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Abstract

In this article, the satisfactory consumption and labor supply elasticities of demand are measured through a model of time allocation that includes eight time assignment equations by using the full time use (the temporal values of the monetary expenditure plus time spent) concept obtained by matching the Classic Family Budget survey with the Time Use survey for Turkey. The cross-sectional data covers the period of 2003–2006 in Turkey. The elasticity results show a clear picture of the relationship between satisfactory consumption and working with commodity demands for Turkey. As a contribution to the literature, we explore the reasons behind the demand for satisfactory consumption through working decisions by measuring well-being inequality for each consumption group. In order to increase the robustness of our result, overall well-being inequality is measured by introducing the axiom of superposed utility of preferences. As expected, overall well-being inequality declines to 0.26, which is 119 percentage points lower than the average rate of well-being inequality (0.57) in Turkey.

Keywords: Time use, life satisfaction, well-being inequality, superposed utilities
JEL classification: C51, D03, J22, I31

Résumé

Dans cet article, les élasticités de consommation satisfaisante et de l'offre de travail de demande sont mesurées par un modèle d'allocation du temps qui comprend huit équations en utilisant du temps complet (les valeurs temporelles des dépenses monétaires plus les dépenses temporelles) obtenu par l'appariement statistique des enquêtes turques sur le Budget des Familles avec l'enquête sur l'Emploi du Temps. Les données transversales couvrent les années 2003-2006 en Turquie. Les résultats des élasticités montrent une image claire de la relation entre la consommation satisfaisante et l'offre du travail avec les demandes de bien pour la Turquie. Comme contribution à la littérature, nous explorons les raisons derrière de la demande de consommation satisfaisante grâce à la décision de travail en mesurant l'inégalité de bien-être dans chaque groupe de consommation. Afin d'augmenter la robustesse de nos résultats, l'inégalité du bien-être général est mesurée en introduisant l'axiome d'utilité superposée de préférences. Comme prévu, l'inégalité de bien-être général diminue à 0,26 , qui est de 119 points de pourcentage moins que le taux moyen de l'inégalité de bien-être général (0,57) en Turquie.

Mots-clés : Allocation du temps, satisfaction de la vie, inégalité du bonheur, utilités superposées
Classification JEL: C51, D03, J22, I31

Introduction

In Becker's (1965) approach to time allocation theory, households are taken into consideration for the first time as production entities that combine time with market goods and transform them into final commodities. These final goods as production are represented in the utility function. An individual optimization program measures the value of time, showing it as the marginal rate of substitution of time for money, allowing individuals to decide time assignment between leisure and working activities. The values of both leisure and work through a consumer optimization program could therefore be estimated by encompassing labor supply and time assignment equations (Jara-Diaz et al., 2013).

However, the proper estimation of time assignment decisions could suffer from certain sources of biases. First, the classification of time as leisure and work or rather, as productive and non-productive, could be misleading since some individuals may engage in and consider certain activities as leisure whereas others view those same activities as chores (Newman et al. 2013). The second limitation would be the fact that the overall well-being of individuals would indeed depend on the level of the simultaneous satisfaction of various activities, not only leisure (Van Praag et al. 2003). The latter issue raises the problem of non-separable preferences in time use.

The objective of this paper is twofold: Firstly, to obtain three new household elasticity values. These are as follows: (i) satisfactory consumption elasticity of demand (ii) labor supply elasticity of demand and (iii) labor supply elasticity of satisfactory consumption which gives information regarding the limit of satisfactory consumption obtained by the working activity of households. As our second objective, once the estimates of our econometric model are obtained for all goods and services, we measure well-being inequality since we will be able to compute individual time use-based utility values for each activity. Well-being inequality values allow us to better understand the reason behind labor supply. Following these objectives, the main theoretical contribution of this paper is to define the axiom of superposed utilities. The importance of this axiom lies in overcoming these two aforementioned problems regarding the conceptualization of time and the inseparability of time use activities when final consumptions are being taken into consideration.

The structure of this paper is as follows: Section 1 begins by presenting the principal features of main time allocation approaches and later will concentrate on the two problems of perception of time and overall satisfaction in economic models. Section 2 begins by outlining the axiom of superposed utilities and later presents the econometric model of time allocation. The details of estimation constraints and the theoretical demonstrations are given at the appendix in order to allow for easier reading. Section 3 describes time use with household budget survey results and also introduces the details of the matching procedure with time valuation of expenditures. Section 4 reports the empirical results and section 5 concludes by focusing on the elasticities of satisfactory consumption and welfare inequalities.

1. Models of time allocation and inherent issues therein

Integrating time assignment decisions into the consumer behavior theory has been explored for more than 40 years through studies from many different perspectives, including those with an interest in either the analysis of leisure time with study of the labor market, or understanding travel behavior, domestic activities and so on. From a theoretical standpoint, when time is considered in the consumer theory, there are three important aspects to be taken into account (Jara-Diaz, 2000): first, its role in utility functions; second, the need to include a time constraint; and third, the need to recognize the relationship between time allocation and goods consumption[2]. The details of the models of the main contributions to the time allocation approach are briefly given in table 1.

Table 1: Main approaches on value of time

Author	Model	Value of Time
Becker (1965)	$MaxU = U(Z_i,...,Z_n) \equiv U(x_i,...,x_n;T_i,...,T_n)$ $\sum_{i=1}^{n} P_i X_i = wW + I_f \rightarrow \lambda$ $\sum_{i=1}^{n} T_i = T - W \rightarrow \mu$	$w = \frac{\mu}{\lambda}$
Johnson (1966)*	$MaxU = U(L,W,G)$ $G = wW \rightarrow \lambda$ $T = L + W \rightarrow \mu$	$\frac{\mu}{\lambda} = w + \frac{\partial U/\partial W}{\lambda} = \frac{\partial U/\partial L}{\lambda}$
Oort (1969)*	$MaxU = U(L,W,t,G)$ $G + c = wW \rightarrow \lambda$ $T = L + W + t \rightarrow \mu$	$-\frac{dU/dt}{\lambda} = w + \frac{\partial U/\partial W}{\lambda} - \frac{\partial U/\partial t}{\lambda}$ $\frac{\mu}{\lambda} = \frac{\partial U/\partial L}{\lambda}$
De Serpa (1971)*	$MaxU = U(x_i,...,x_n;T_i,...,T_n)$ $\sum_{i=1}^{n} P_i X_i = I_f \rightarrow \lambda$ $\sum_{i=1}^{n} T_i = T \rightarrow \mu$ $T_i \geq a_i X_i \rightarrow \kappa_i$	$\frac{\mu}{\lambda} = \frac{\partial U/\partial L}{\lambda}$ $\frac{\kappa_i}{\lambda} = \frac{\mu}{\lambda} - \frac{\partial U/\partial T_i}{\lambda}$
Evans (1972)*	$MaxU = U(T_i,...,T_n)$ $\sum_{i=1}^{n} wT_i \geq 0 \rightarrow \lambda$ $T - \sum_{i=1}^{n} T_i = 0 \rightarrow \mu$ $T_i - \sum_{\forall j \neq 1}^{n} b_{ij} T_j \geq 0 \rightarrow \kappa_i$	$\frac{\mu}{\lambda} = \frac{\partial U/\partial L}{\lambda} + w_L$ $\frac{\kappa_i}{\lambda} = \frac{\mu}{\lambda} - \frac{\partial U/\partial T_i}{\lambda} - w_i$
Small (1982)*	$MaxU = U(L,W,s,G)$ $G + c(s) = I_f + wW \rightarrow \lambda$ $L + t(s) = T - W \rightarrow \mu$ $F(s,W;w) = 0 \rightarrow v$	$\frac{\mu}{\lambda} = w + \frac{\partial U/\partial W}{\lambda} - v\frac{\partial F/\partial W}{\lambda}$
Gronau (1986)*	$MaxU = U(Z_i,...,Z_n,Z_W)$ $\sum_{i=1}^{n} P_i X_i + P_W X_W = I(Z_W) + I_f \rightarrow \lambda$ $\sum_{i=1}^{n} T_i + W = T \rightarrow \mu$ $Z_i = f_i(X_i,T_i)$ $Z_W = f_W(X_W,W)$	$\frac{\mu}{\lambda} = w + \frac{\partial U/\partial W}{\lambda} - P_W \frac{\partial X_W}{\partial W}$ $Z_W = W$ $I(Z_W) = wW$
Jara-Diaz et al.(2013)	$MaxU = U(T,X)$ $I + wW - \sum_j P_j X_j \geq 0 \rightarrow \lambda$ $T - W - \sum_i T_i = 0 \rightarrow \mu$ $T_i - T_i^{Min} \geq 0 \rightarrow \kappa_i \quad \forall i$ $X_i - X_i^{Min} \geq 0 \rightarrow \eta_i \quad \forall j$	$\frac{\kappa_i}{\lambda} = \frac{\mu}{\lambda} - \frac{\partial U/\partial T_i}{\lambda}$ $\frac{\mu}{\lambda} = w + \frac{\partial U/\partial T_w}{\lambda}$
Gardes (2014)	$MaxU = U(Z_1,...,Z_i)$ $Z_i = f_i(x_1,x_2,\dots x_i;t_1,t_2,\dots,t_i)$ $\left.\begin{array}{l} \sum_i p_i x_i = Y \\ \sum_i T_i x + W = T \\ Y = wW + V \\ T_i = x_i \tau_i \end{array}\right\} px + \sum_i w x_i \tau_i = wT + V$	$\pi_i = \frac{x_i p_i + x_i w \tau_i}{x_i p_i}$ $w = \frac{p_i(\pi_i - 1)}{\tau_i}$

**source:* Jara-Diaz (1999)

Glossary for the variables is given at Table A2 at Appendix II

[2]i.e. The utility function depends on consumption, and consumption means expenses; therefore more work is taken into consideration with increasing income, and that consumption requires time.

Recent developments in time use point out two problematic areas regarding these models. However, defining and measuring time use within an individual optimization program would suffer from two issues:

Firstly, the difficulty lies in the fact that the classification of time by the neoclassical theory would only be two such working and leisure (i.e. non-working). The main criterion is the emotional counterparts of any activity. All painful activities are considered as working, while others are assumed to be leisure since they are pleasant[3]. Therefore, the classification of time as leisure or working would not be easy since the social and psychological determinants shaping individual perception about time use; hence the qualification and the quantification activities are highly subjective (Kleiber et al. 2011, Neulinger, 1974). In fact, recent research points out that even if leisure is a key component of life and a core ingredient for overall well-being, it is more convenient to differentiate it into two distinct structures (Newman et al., 2013): structural and subjective leisure (Kelly and Godbey, 1992; Lloyd and Auld 2002; Brajsa Zganec et al. 2011). Structural/Subjective leisure perception is simply the amount of activities/time spent outside obligated work time, while the second is accepted as an engagement in leisure as subjectively defined by individuals. The key difference between structural and subjective leisure is whether leisure is externally or internally defined. The main idea that can be gleaned from leisure distinction is that an activity can be accepted as part of leisure time, and hence can satisfy some individuals while not satisfying others. Therefore, the common thought is that leisure activities and similar recreation satisfaction have even been shown to be greater predictors of life satisfaction and quality of life than income, sex, education, religiosity, marital status, age, health or employment status (Riddick, 1985).

Viewed from a theoretical standpoint, such a description of leisure time as the sole determinant or a proxy of satisfactory decisions and life quality would be misleading when consumption decisions are taken into consideration. In fact, no matter what type of action a decision-maker takes, working or non-working, time use and consumption are continuously combined. In other words, we cannot think about time without consumption or consumption without time. Therefore, the opportunity cost of time is independent from the scheduling of daily activities, the type of activities and there is no difference between time and monetary budget choices. That is to say, individuals always choose to allocate these budgets collectively. The reason why, as was proposed by Van Praag et al.(2002) and Van Praag and Ferrer-i-Carbonell (2007) a two-layer approach to life satisfaction, a general satisfaction of an individual would depend not only on leisure but also on other subjective domain satisfactions gathered from job, financial, housing, health and environmental facts.

Consumption-based satisfactory leisure of life

These findings together suggest that well-being or overall satisfaction is indeed a behavioral phenomenon, implying on optimum combination of time and commodity, and its value would depend on the simultaneous satisfaction derived from these activities. However, these issues necessitate distinguishing the perception of leisure time from that assumed by the theory. We suppose defining the concept of *consumption-based satisfactory leisure of life* by arguing that consumptions are indeed assumed to be just as pleasant, so long as they are used

[3] Thereby, Marginalists' exchange value assumes that the sentimental background of an undesirable working activity only is the pain which is always compensated by equivalent pleasant consumption in the future.

for protecting and improving the actual or expected quality of life (Gunes, 2014)[4]. At that point, three additional hypothetical distinctions regarding standard theory had to be made[5]:

(i) Each consumption decision determines a specific period of time that consists of satisfactory leisure time owing to *not feeling hungry, being healthy, not-transportation*…etc. This corollary is equal to saying that consumption-based satisfactory leisure time values for each activity also determine the level of life satisfaction for individuals.

(ii) Satisfactory leisure time value is the time period that passes between two similar consumption moments. Every consumption activity determines the amount of time as a unit until the next consumption moment arrives. During this time, other achieved consumption activities, one inside the other, will give their own units of time and so on. In this sense, working time is not different from non-working time since both are part of these satisfactory leisure times.

(iii) Likewise, total satisfactory time is continuous and given by superposition of these satisfactory time intervals. In this sense, the utilities of satisfactory leisure times are dependent on each other and successively connected to previous ones. The total utility could be measured through superposed utilities.

1. Theory and econometric model

Axiom of superposed utilities[6]

Consumption of the economic goods $x \in M$ creates satisfactory time values *T*. The utility function *U(.)* with *U*: $(x,T) \to \mathbb{R}_+^M$. Suppose two consumption goods x_1 and x_2 would simultaneously be consumable. Thus, $U(x_1,T_1;x_2,T_2)$ and $U:(x_1,T_1)\times(x_2,T_2)\to\mathbb{R}_+^M$ with both $u_1:(x_1,T_1)\to\mathbb{R}_+^M$ and $u_2:(x_2,T_2)\to\mathbb{R}_+^M$. Utilities are superposed if the consumer prefers to consume both goods simultaneously since

$$U(x_1,T_1;x_2,T_2) \geq u_1(x_1,T_1)+u_2(x_2,T_2) \tag{1}$$

The superposed utility axiom of inequality (1) does not violate the initial position axioms of choice, transitivity and the addition rules of rationality assumptions of neoclassical consumer preferences properties. The axiom given in (1) reveals that an individual simultaneously profits, each moment of life, from his already satisfied needs.

Econometric model

Let *T*, *w*, ω, *wT*, t^w, t_i^c, t_i^a and *i* respectively be the total time, hourly wage, opportunity cost for time, full income, working time, consumption time, leisure time and activity index *(i=1,2,…n)*

Supposing that $T_i^a = \sum_i^n t_i^a$ is the total leisure time where $T_i^a = T$ with

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[4] Satisfactory leisure time is created by the necessary action combining time and commodity to satisfy needs. The logic underpinning this idea is that it is not possible to move from an undesirable painful state to desirable pleasant state without an action,while the inverse of this statement is not true.

[5] Theoretical and mathematical proof of these hypotheses are given at Appendix I.

[6] For the detail see Appendix I – 1.1.

$$T = T_i^a = t^w + \sum_{i=1}^{n-1} t_i^c \tag{2}$$

defined for the domain $D = \{(t^w, t_i^c) \mid t^w > 0, t_i^c > 0\}$ with range of $R = \{t_i^a \mid T^a(t^w, t_i^c) > 0\}$. Each t_i^a in $\mathbf{T}$ is measured by summing the consumption activity elements defined in the consumption vector ($t_i^c \in \mathbf{T}$), and the working activity vector ($t^w \in \mathbf{T}$). *n-1* denotes the exempted consumption activity which wanted to be measured. For instance, let t_{food}^c the time spent for food eating activity, the satisfactory time interval owing to not being hungry (t_{food}^a) is equal to the sum of time spent activities other than food. The equation (2) implies that (t_{food}^a) is simply the time interval that passes between two food consumption moments, thus it must be excluded from the rest of the consumption activities. Aggregate leisure time for all activities is equal to

$$T_i^a = T^w + T_i^c \tag{2.1}$$

Following the methodology proposed by Gardes (2014) the full budget constraint (for $w \neq \omega$) is $p_i x_i + \omega t_i^c = w T_i^a$ or $p_i x_i = w T^w - (\omega - w) T_i^c$. The price of time could be measured by following the consumer program

$$\begin{aligned}
&Max U(t^w, t^c, X) \\
&p_i x_i - w T^w + (\omega - w) T_i^c = 0 \rightarrow \lambda \\
&t^w + \sum_{i=1}^{n-1} t_i^c - T_i^a = 0 \rightarrow \mu
\end{aligned}$$

by assuming that a common tangent line exists with parallel gradient vectors for each curve. Thus, the program could be solved by $\nabla U = \lambda \nabla f_Y + \mu \nabla f_T$ with the scalar values λ and μ. In terms of a Lagrange function, the program then could be written as

$$L = U(t^w, t^c, X) - \lambda(p_i x_i - w T^w + (\omega - w) T^c) - \mu(t^w + \sum_{i=1}^{n-1} t_i^c - T_i^a)$$

The ratio μ / λ is a marginal rate of substitution (MRS) between time and money as the value of time. The value of time assigned to consumption and working activity is $(\partial U(.) / \partial T_i^c) / \lambda$ and $(\partial U(.) / \partial T^w) / \lambda$ respectively. The first order conditions for t^w, t^c, X gives

$$\frac{\mu}{\lambda} = w - \omega + \frac{U'_{T^c}}{\lambda} = w + \frac{U'_{T^w}}{\lambda} \tag{2.2}$$

where $\omega = \dfrac{U'_{T^c} - U'_{T^w}}{\lambda}$ and

$$\frac{\omega}{p} = \frac{U'_{T^c} - U'_{T^w}}{U'_x} = \frac{\partial X_i}{\partial T_i^c} - \frac{\partial X_i}{\partial T^w} \tag{2.3}$$

Equation (2.3) determines the opportunity cost of time given by the difference between the total effect on the consumption of goods, of the change in consumption and working times.

First order condition

By multiplying X_i with both sides of the equation (2) we obtain

$$X_i T_i^a = X_i t^w + X_i \sum_{i=1}^{n-1} t_i^c \tag{3}$$

Eq. (3) is the detailed version of DeSerpas' postulation on Beckerian utility function where the $Z_i = f(T_i^a, X_i)$ or further as $Z_i = f(T^w, T_i^c X_i)$; hence the utility function is $U = U(Z_i) = U(T^w, T_i^c X_i)$. More generally, the satisfactory action implies maximizing the utility function depending on the optimal time allocation between working and consuming. To turn the preceding framework into an analytically workable model, let U(.) given a Cobb–Douglas form(see Gardes, 2013 and Jara-Díaz, 2013).

$$U = \Omega \prod T_w^{\alpha} \prod_i T_{c_i}^{\beta_i} \prod_i X_i^{\eta_i} \tag{4}$$

Ω measures individual the time storage capacity of satisfactory consumption metabolism (see Simon, 1956); α, β_i, η_i are the exponents of working, consumption activity and amount of consumption goods for commodity *i* respectively. Our Lagrangian consumer program above then could be rewritten as

$$L = U(\Omega \prod T_w^{\alpha} \prod_i T_{c_i}^{\beta_i} \prod_i X_i^{\eta_i}) - \lambda(p_i x_i - wT^w + (\omega - w)T_i^c) - \mu(t^w + \sum_{i=1}^{n-1} t_i^c - T_i^a)$$

The first order condition given in the solution (2.3) becomes

$$p_i X_i^* = \frac{\eta \omega_h T^{w*} T_i^{c*}}{\beta T^{w*} - \alpha T_i^{c*}} \tag{5}$$

Rearranging the equation (5) subject to the full budget constraint yields

$$\frac{T_i^{a*}}{T} = \frac{\omega}{w} \frac{T_i^{c*}}{T} + \frac{\omega}{w} \frac{T_i^{c*}}{T} \left(\frac{\eta T^{w*}}{\beta T^{w*} - \alpha T_i^{c*}} \right) \tag{6}$$

which is equal to

$$\frac{T_i^a}{T} = \frac{\omega_h}{wT} T_i^{c*} + \frac{\omega_h}{wT} \left(\frac{\beta}{\eta} \frac{1}{T_i^{c*}} - \frac{\alpha}{\eta} \frac{1}{T^{w*}} \right)^{-1} \tag{6.1}$$

Let $\kappa_i = T_i^{a*} / T$ and $\psi = \omega_h / wT$, the equation (6.1) could be simplified as

$$\kappa_i = \psi_i T_i^{c*} + \psi_i \left(\frac{\beta}{\eta} \frac{1}{T_i^{c*}} - \frac{\alpha}{\eta} \frac{1}{T^{w*}} \right)^{-1} \tag{6.2}$$

The population regression function is

$$\kappa_i = \delta_0 + \delta_1 \psi_i T_i^{c*} + \delta_2 \psi_i T_i^{c*} T^{w*} \left(\frac{1}{\delta_3 T^{w*} - \delta_4 T_i^{c*}} \right) + u_i \tag{7}$$

Where $\delta_3 = \beta/\eta$ and $\delta_4 = \alpha/\eta$ with the constraints[7] of $\delta_3 + \delta_4 = 1$ and $\delta_3, \delta_4 > 0$. Therefore the stochastic variables from equation (7) determines the equation (2.3) using (4) as

$$\frac{\partial X_i}{\partial T_i^c} - \frac{\partial X_i}{\partial T^w} = \hat{\delta}_3 \frac{X}{T_i^c} - \hat{\delta}_4 \frac{X}{T^w} \tag{8}$$

The equation (8) defines (*i*) the consumption time elasticity of demand and (*ii*) labor supply elasticity of demand. By substituting (*ii*) into (*i*) we can solve the labor supply elasticity of satisfactory consumption (*iii*) as

$$\left.\begin{array}{ll} (i) & \hat{\delta}_3 = \dfrac{\partial X_i}{\partial T_i^c} \dfrac{T_i^c}{X} = \dfrac{\beta}{\eta} \\ (ii) & \hat{\delta}_4 = \dfrac{\partial X_i}{\partial T^w} \dfrac{T^w}{X} = \dfrac{\alpha}{\eta} \end{array}\right\} \quad (iii) \quad \frac{\hat{\delta}_3}{\hat{\delta}_4} = \frac{\partial T^w}{\partial T_i^c} \frac{T_i^c}{T^w} = \frac{\beta}{\alpha} \tag{8.1}$$

More precisely, (*i*) measures the effect of change of satisfactory consumption on consumer demand. High demand elasticities are caused by low level satisfaction. These values measure the degree of displeasure felt from lack of satisfaction of needs. Thus, it is expected that elasticity values draw closer to zero (one) when the satisfaction (dissatisfaction) obtained from consumption increases. Therefore, (*ii*) informs us of working motivation. That is to say, it measures the intensity of the consumption motivation behind working decisions. In other words, which consumption groups individuals are more inclined to work. Higher elasticity values signify more intensive working tendencies and *vice versa*. Finally, the level of satisfactory consumption for a given labor supply is measured by (*iii*). These values provide us with the level of satisfaction provided by working activity. Following (*i*) and (*ii*), lower (higher) elasticities for (*iii*) signify more (less) needs met by work; hence by income.

2. Dataset and temporal value of consumption

We combine the monetary and time expenditures into a unique consumption activity at the individual level. We proceed with the matching of these surveys by regression on similar exogenous characteristics in both datasets as age, matrimonial situation, possession of cell phone, home ownership, number of household members, geographical location separately for head of household and wife[8].

More precisely, we estimate 8 types of time use at Time Use Survey (TUS) which are also compatible with the available data from Household Budget Survey (HBS) as follows: Food Time (TUS) with Food Expenditures (HBS); Personal Care and Health Time (TUS) with Personal Care and Health Expenditures (HBS); Housing Time (TUS) with Housing Expenses (HBS); Clothing Time (TUS) with Clothing Expenditures (HBS); Education Time (TUS) with

[7] The estimation restrictions and the derivation of total utility function are given in Appendix II and Appendix III respectively. For the theoretical and mathematical proof of these analyses see Gunes (2015).

[8] The selection equation concerns the households which have a positive time use of their activities.

Education Expenditures (HBS); Transport Time (TUS) with Transport Expenditures (HBS); Leisure Time (TUS) with Leisure Expenditures (HBS); Other Time (TUS) with Other Expenditures (HBS)

The food Time consists only of cooking because it is not possible to separate eating activity from Personal Care in the time use survey. Care time consists of personal care, commercial-managerial-personal services, helping sick or old household person and eating activity. Housing Time corresponds to household-family care as home care, gardening and pet animal care, replacement of house-constructional work, repairing and administration of household. Clothing Time consists of washing clothes and ironing. Education Time includes study (education) and childcare. Transport Time consists of travel and unspecified time use. Leisure Time corresponds to voluntary work and meetings, social life and entertainment such as culture, resting during holiday, sport activities, hunting, fishing, hobbies and games, mass media like reading, TV/Video, radio and music. Other Time includes employment and labor searching times.

Valuation of time

By taking advantage of the available TUS and HBS data, we are able to choose the opportunity cost method. Two possible opportunity cost methods for the time valuation of consumption can be used: the first method dictates that we impute the wages net of taxes for non-working individuals using the two-step Heckman procedure: supposing that only time use is perfectly exchangeable for non-market and market activities. In this method the first step estimates a probit equation for participation. The natural logarithm of monthly income, age, age-squared, education dummies, urban variables with the explanatory variables of couples, number of children and number of household members are used to predict the underlying wage rate of households that do not work. Thus, in the second stage the opportunity cost of non-market work is estimated as the expected hourly wage rate in the labor market for those men and women not working[9]. The second method refers to sole market substitution which consists of imputing the same hourly rate for all individuals represented in the surveys, namely minimum wage rate for Turkey in the years 2003, 2004, 2005 and 2006.

Given that individuals spend their time in domestic activities and that time has a cost, we consider the total time of the households as the sum of the temporal values of their monetary expenditure plus time spent for each activity. To do so, we choose the methodology of dividing monetary earnings by minimum wages for the working individuals, while we use the opportunity cost of time obtained through the two-step Heckman procedure for the monetary incomes of non-working individuals. The opportunity cost may rather be between these two values (see the discussion in Gardes, Starzec, 2014). The purpose of this double recovery is to obtain more accurate total time values for working and non-working individuals and to increase reliability of the following results.

3. Results

Demand elasticities of satisfactory consumption

The satisfactory consumption time elasticity of demand and labor supply elasticity of demand are obtained from a system of time assignment equations using the Generalized Method of Moments (GMM) by regrouping time activities and expenditures in eight different

[9] For Heckman two step selection model estimation results in 2006 see Table A3 in the Appendix II.

commodities for Turkey. The model's estimation of full time (i.e. equal to the temporal values of monetary expenditures plus time values of domestic activities) expenditure from the pooled cross-sectional data covering the period of investigation 2003-2006, is presented in **Table A4**. The size of the pooled sample increases to 34 357 households. We minimize the problem of quality effects due to the difference in time allocation to the same activity by different individuals, by including the control variables in the model: level of home ownership, number of rooms in the house, summer homeownership, geographic environment (urban or rural), education level of head of household and durable goods dummies such as computer ownership, car ownership, having use of a good heating system, television ownership, access to the internet, a refrigerator, deep freezer, dishwasher, oven, cablecast etc. Over-identifying restrictions in the estimation is 0.43 with a degree of freedom equal to 26. Chi-square p value for monetary estimations is 0.80, which is bigger than 0.05 where no hypotheses and the validity of the identifying instruments cannot be rejected for the chosen control variables.

We calculate the labor supply elasticity of satisfactory consumption both for whole population and for different family types (with and without children). The results are presented in **Table 2**.

Table 2: Elasticity results for the years between 2003 and 2006

	Familly with children			**Familly without children**			**Whole population**		
Commodity Groups	$\hat{\delta}_3$	$\hat{\delta}_4$	$\hat{\delta}_3/\hat{\delta}_4$	$\hat{\delta}_3$	$\hat{\delta}_4$	$\hat{\delta}_3/\hat{\delta}_4$	$\hat{\delta}_3$	$\hat{\delta}_4$	$\hat{\delta}_3/\hat{\delta}_4$
Food	0,418 *** (0,012)	0,582 *** (0,012)	0,718	0,434 *** (0,026)	0,566 *** (0,070)	0,768	**0,111** *** (0,000)	**0,889** *** (0,001)	**0,125**
Personal Care+Health	0,517 *** (0,000)	0,483 *** (0,000)	1,068	1,000 *** (0,125)	0,000 (0)	-	**0,513** *** (0,000)	**0,487** *** (0,000)	**1,055**
Housing	0,343 *** (0,001)	0,657 *** (0,003)	0,523	0,523 *** (0,033)	0,477 *** (0,026)	1,094	**0,525** *** (0,026)	**0,475** *** (0,031)	**1,105**
Clothing	0,260 *** (0,000)	0,740 *** (0,004)	0,351	0,357 *** (0,018)	0,643 *** (0,029)	0,556	**0,042** *** (0,000)	**0,958** *** (0,000)	**0,044**
Education	0,343 *** (0,002)	0,657 *** (0,002)	0,521	0,461 *** (0,046)	0,539 *** (0,060)	0,855	**0,392** *** (0,000)	**0,608** *** (0,000)	**0,644**
Transport	0,508 *** (0,052)	0,492 *** (0,019)	1,032	0,388 *** (0,012)	0,612 *** (0,035)	0,634	**0,564** *** (0,009)	**0,436** *** (0,004)	**1,293**
Leisure	0,577 *** (0,000)	0,423 *** (0,000)	1,361	0,604 *** (0,000)	0,396 *** (0,000)	1,525	**0,728** *** (0,001)	**0,272** *** (0,000)	**2,676**
Others	0,709 *** (0,012)	0,291 *** (0,012)	2,441	0,518 (0,348)	0,482 (0,321)	1,076	**0,514** *** (0,000)	**0,486** *** (0,000)	**1,059**

***p<0.01, **p<0.05, *p<0.1

Sources: Household Budget Survey (2003, 2004, 2005, 2006) and Time Use Survey (2006)

$\hat{\delta}_3$: Satisfactory consumption elasticity of demand

$\hat{\delta}_4$: Labor supply elasticity of demand

$\hat{\delta}_3/\hat{\delta}_4$: Labor supply elasticity of satisfactory consumption

The contribution of identifying the working elasticities of satisfactory consumption in this way, is to give us an idea of the satisfaction level provided by working activities for surrounding households. The first two columns of **Table 2** shows each population respectively represent the satisfactory consumption time elasticity of demand and labor supply elasticity of demand, while the last third column underneath illustrates working elasticities of satisfactory consumption. The first column indicates that individuals are more satisfied with food and clothing. Therefore, high demand elasticity results show that individuals are less satisfied for health with personal care, housing, others, education, transport and leisure, respectively. It is worth noting that at the population level, food is a necessary good, as anticipated by the theory, and leisure is generally considered a luxury good. In the second column, as an affirmation of the first one, it allows us to discover for the sake of which consumption group households are more likely to engage in work. Increasing demand for food and clothing are the top two reasons behind working decisions, while leisure took the last place. This also raises the question of whether a working activity can meet the very different satisfaction of different consumptions. The answer is given in the last column. These elasticities show us the level of satisfaction obtained through working activities. Health with personal care, housing and others have a unitary demand. An increase (decrease) in working activity works to generate a proportional increase (decrease) of satisfactory consumption demand for these groups of goods. Therefore, leisure, education and transportation are elastic since an increase in work generates greater satisfactory consumption for these groups. However, food and clothing have the most satisfactory consumptions, therefore, less elastic

Well-being inequality

Therefore, we are still not able to grasp the underlying logic behind the working decision because elasticity results only show a picture of the relationship between satisfactory consumption and working with commodity demands. Since one of the objectives of this paper is to analyze the overall satisfaction through superposed utilities (i.e. superimposing satisfaction waves), we propose measuring satisfaction inequity for each and total utility in order to better understand households' working decisions. In the literature, there are the usual income inequality measures such as the Gini-index, Range, Range-ratio, the McLoone-Index, the Coefficient of Variation, the Theil-index, the Pareto-index, the Atkinson-index or Subjective well-being index (Van Praag, 2011). Each index has its own advantages and disadvantages. Generally speaking, most of these indexes suffer from ignoring all but two of the observations, they do not weight observations, are affected by inflation, skewed by outliers, ignore values above the median, show no intuitive motivating picture, cannot directly compare populations with different sizes or group structures or are comparatively mathematically complex. We choose the Gini Index which is generally regarded as the gold standard in economic work, which allows us to incorporate all data and make a direct comparison between units with different size populations. Furthermore, we can assume that the parameters already include the information regarding the reference group effect on inequality since we observe consumption and time use preferences of households by employing durable and non-durable commodity control variables.

Well-being inequality results are given under the **Table 3**. According to whole population results, inequality takes the value range from 0.47 to 0.81 while it is from 0.47 to 0.80 and from 0.48 to 0.72 respectively for families with and without children.

Table 3: Well-being inequality results by Gini-index (2003-2006)

Commodity Groups	With children	Without chilren	All Population
Food	0,47	0,48	0,47
Personal Care+Health	0,54	0,50	0,50
Housing	0,50	0,48	0,49
Clothing	0,80	0,72	0,81
Education	0,51	0,65	0,66
Transport	0,56	0,60	0,60
Leisure	0,47	0,51	0,51
Others	0,55	0,51	0,53
Total (Total utility inequality)	**0,23**	**0,24**	**0,26**

Lowest inequality would have been felt from food consumption, while clothing is the highest source of inequality. This result deserves particular attention. In fact, as it can be followed from the working elasticity of demand, this result justifies higher working tendencies for the clothing demand. The idea is that, high level well-being inequality felt from clothing consumption is the reason behind an increase in working activity. Higher working tendencies are later followed by education, transportation, others, leisure, personal care with health and housing. Thus two necessities, food and housing, took the lowest inequality results. Additionally, the total utility-based well-being inequality is 0.26 for these years. The main idea that is to be gleaned from this result is that superposing utilities enable the reduction of the well-being inequality by combining different consumptions at the same time. For example, the satisfaction obtained by food consumption at home during a leisure activity would be expected to decrease well-being inequality in Turkey.

4. Conclusion

We estimate satisfactory consumption and labor supply elasticities of demand on micro cross-sectional data within a model of time allocation that includes eight time assignment equations by using the full time use (the temporal values of their monetary expenditure plus time spent for each activity) concept obtained by matching the Classic Family Budget with the Time Use surveys for Turkey.

As the main contribution of this study, we compute, firstly, the level of satisfaction obtained from working activity for each commodity group and estimate their effect on household demand. These results show a picture of the relationship between satisfactory consumption and working with commodity demands. Secondly, we incline to understanding the reasons behind the demand for satisfactory consumption through working decisions by measuring well-being inequality for each consumption group. To increase the robustness of our result we also compute overall well-being inequality by introducing the axiom of superposed utility of preferences. Such an axiom is necessary for two reasons: first, the classification of time is highly subjective and second, the satisfactions obtained from each activity are internally connected. Therefore, the latter issue raises the problem of non-separable preferences in time use since different goods and time spent are simultaneously combined.

The model is well estimated with almost all parameters being significant. Working activity enables higher satisfactory consumption for food, clothing, health with personal care, housing and others, respectively. Therefore, leisure and transportation are less satisfied consumption groups for the given level of labor force. However,we have shown that, having a high level of satisfaction requires a higher level of working hours due to a state of well-being inequality. In other words, high level well-being inequalities are respectively felt from clothing (0.81), education (0.66), transportation (0.60), others (0.53), leisure (0.51), personal care with health (0.50), housing (0.49) and food (0.47). Thus two necessities, food and housing, took the lowest inequality results. Our results are coherent with the income and price elasticities with formal and informal working tendency results pointed out by Aktuna-Gunes et al. (2015). Therefore, following the methodology proposed in this study, we also compute overall well-being inequality through superposed utilities. When simultaneous satisfactory consumptions are being considered, total well-being inequality declines to 0.26 which is 119 percentage points lower than average well-being inequality, at 0.57.

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APPENDIX I

Theoretical Model

As proposed by Gunes (2014), each consumption decision determines a specific time period that consists of satisfactory moments of *not feeling hungry, being healthy, non-transportation*…etc. Every consumption activity creates satisfactory time as a unit until the next consumption moment arrives[10]. Specific satisfactory time values correspond to the time period that passes between two similar consumption moments. Supposing that an individual possesses M quantities of economic goods x_m, $m=1,2,\ldots,M$, classified within N consumption groups x_n, $n=1,2,\ldots,N$, so that $x_{11}\ldots x_{MN}$.

Proposition 1: Total consumption-based satisfactory time values would be obtained by adding up the temporal algebraic vectors.

Proof 1: Let the time that passes between the last two consumption moments $t–1$ and t for similar consumption goods to be defined by the operation $T_{mn} = x_{m_{t-1}n} - x_{m_t n}$. The time vector $\mathbf{T}$ comprise of the satisfactory time between the last two consumptions of similar x in M. The position of this individual is given by the vector $\mathbf{T}$ which can be carried out with the origin of the M dimensional space in a rectangular coordinate system at the point $\left(T_{11}T_{21}\cdots T_{12}T_{22}\cdots T_{MN}\right)$. Thus, the total consumption-based satisfactory time values for each consumption group could be obtained by following the equation

$$T = \left\langle T_{11} + T_{21} + \cdots + T_{M1}; T_{12} + T_{22} + \cdots + T_{M2}; T_{1N} + T_{2N} + \cdots + T_{MN} \right\rangle \tag{9}$$

Proposition 2: The temporal order between consumption moments could be calculated by subtracting each satisfactory time from the total time.

Proof 2: The length of total time is then equal to[11]

$$\left|T\right| = \sqrt{\left(T_{11} + T_{21} + \cdots + T_{M1}\right)^2 + \left(T_{12} + T_{22} + \cdots + T_{M2}\right)^2 + \cdots + \left(T_{1N} + T_{2N} + \cdots + T_{MN}\right)^2} \tag{10}$$

Each consumption moment could be measured by a scalar value $\left|T\right| - \left|T_{MN}\right|$ and bigger differences imply older consumptions. Let $T_{1n} \in \mathbf{T}$ and $T_{2n} \in \mathbf{T}$ are the two different satisfactory time values created by the consumption the goods x_{1n} and x_{2n}, respectively. Herein, there would only be three possible temporal orders between these consumption-based satisfactory time values at a given point of time (algebraic denotations are shown in parentheses):

1. Individual consumes x_{1n} before x_{2n}; ($\left|T_{1n}\right| < \left|T_{2n}\right|$)
2. Individual consumes x_{1n} after x_{2n};($\left|T_{1n}\right| > \left|T_{2n}\right|$)
3. Individual consumes x_{1n} and x_{2n} at the same; ($\left|T_{1n}\right| = \left|T_{2n}\right|$)

[10] The logic underpinning this idea is that it is not possible to move from an undesirable painful state to desirable pleasant state without an action, while the inverse of this statement is not true.
[11] During this time, other achieved consumption activities, one inside the other, will give their own units of time and so on. In this sense, working time is not different from non-working time since both are part of these satisfactory times.

The consumption-based satisfactory time values are additive in a continuous order. More precisely, if temporal order refers to consumption case 1 or 2, another satisfactory consumption time interval would have to exist, seen between T_{1n} and T_{2n}. Assuming the third time interval is $T_{3n} \in \mathbf{T}$, therefore $|T_{1n}| - |T_{2n}| = T_{3n}$ with $T_{3n} \geq 0$. Thus,temporal order 1 would in turn imply three order possibilities[12]:

4. $|T_{1n}| < |T_{3n}| < |T_{2n}|$
5. $|T_{1n}| < |T_{3n}| = |T_{2n}|$
6. $|T_{1n}| = |T_{3n}| < |T_{2n}|$

1.1. Axioms of choice theory

Following the methodology proposed by Frish (1957), the choice of the individual is not such that every infinitesimal movement around position **T** is zero choice. Supposing that **u** is a direction vector directed along the normal to L_μ, L_μ is a linear variety with μ extended dimensions in M-dimensional space (μ<M). Let $\partial\mathbf{T}$ be an infinitesimal movement around **T**. Herein we see that the possible values of $\mathbf{u}\partial\mathbf{T}$ are distributed around **T** in a similar way to the preferences of the individual. More precisely, if $\partial_1\mathbf{T}$ and $\partial_2\mathbf{T}$ are two infinitesimal movements around **T**, then the choice of the individual would be defined within

$$\left(\mathbf{T}\partial_1\mathbf{T}\right) \gtreqless \left(\mathbf{T}\partial_2\mathbf{T}\right) \tag{11}$$

due to the fact that

$$\mathbf{u}\partial_1\mathbf{T} \gtreqless \mathbf{u}\partial_2\mathbf{T} \tag{12}$$

The product of $\mathbf{u}\partial\mathbf{T}$ defines the utility of the movement $\partial\mathbf{T}$ around fixed point **T**[13]. Therefore, the simultaneous satisfactory consumption possibility would imply two extra cases for inequalities (11) and (12). Let an infinitesimal movement of $\partial_3\mathbf{T}$ be defined under the utility of this movement $\mathbf{u}\partial_3\mathbf{T}$, thus

$$\left(\mathbf{T}\partial_1\mathbf{T}\right) < \left(\mathbf{T}\partial_3\mathbf{T}\right) + \left(\mathbf{T}\partial_2\mathbf{T}\right) \tag{13}$$

since

$$\mathbf{u}\partial_1\mathbf{T} < \mathbf{u}\partial_3\mathbf{T} + \mathbf{u}\partial_2\mathbf{T} \tag{14}$$

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[12] These conditions for the order case 2 imply $|T_{1n}| > |T_{3n}| > |T_{2n}|$, $|T_{1n}| = |T_{3n}| > |T_{2n}|$ and $|T_{1n}| > |T_{3n}| = |T_{2n}|$.

[13] The values of the components u_1, u_2,.. u_M of **u** explained are marginal utility values of the good 1,2,…M at point **T.** Thus we have the direction of maximum values but not the size of these marginal utilities. Thus, in plan L, $\mathbf{u}\partial\mathbf{T} = \mathbf{0}$, which is equal to $u_1\partial T_{1n} + u_2\partial T_{2n} + \ldots + u_{MN}\partial T_{MN} = 0$.

Or

$$\left(\mathbf{T}\partial_1\mathbf{T}\right)+\left(\mathbf{T}\partial_3\mathbf{T}\right)>\left(\mathbf{T}\partial_2\mathbf{T}\right) \tag{15}$$

due to this

$$\mathbf{u}\partial_1\mathbf{T}+\mathbf{u}\partial_3\mathbf{T}>\mathbf{u}\partial_2\mathbf{T} \tag{16}$$

The right hand side of the inequality (14) and left hand side of inequality (16) are shown by the displacement vectors $\overrightarrow{\mathrm{u}\partial_3\mathrm{T}\mathrm{u}\partial_2\mathrm{T}}$ and $\overrightarrow{\mathrm{u}\partial_1\mathrm{T}\mathrm{u}\partial_3\mathrm{T}}$, respectively, starting from the fixed point **T**. These effects, combined called total utility, consist of the sum of simultaneous movements given by these displacement vectors.

Definition 1 (*Axiom of superposed utilities*): All movements of $\partial\mathbf{T}$ necessitate consumption of economic goods $x\in M$ and create satisfactory time values *T*. The utility function *U(.)* with *U*: $(x,T)\to\mathbb{R}_+^M$. Suppose two consumption goods x_1 and x_2 would simultaneously be consumable. Thus, $U(x_1,T_1;x_2,T_2)$ and $U:\left(x_1,T_1\right)\times\left(x_2,T_2\right)\to\mathbb{R}_+^M$ with both $u_1:\left(x_1,T_1\right)\to\mathbb{R}_+^M$ and $u_2:\left(x_2,T_2\right)\to\mathbb{R}_+^M$. Utilities are superposed if the consumer prefers to consume both goods simultaneously since $U\left(x_1,T_1;x_2,T_2\right)\geq u_1\left(x_1,T_1\right)+u_2\left(x_2,T_2\right)$.

The superposed utility axiom of this inequality does not violate the initial position axioms of choice, transitivity and the addition rules of rationality assumptions of neoclassical consumer preferences properties:

<u>Axiom of choice</u>: The position of an individual is given by $\mathbf{t}\in\mathbf{T}$ and has two displacement possibilities such as $\mathbf{y}\in\mathbf{T}$ and $\mathbf{h}\in\mathbf{T}$. There are three consumption (or exchange) possibilities for the commodities x_t,x_y,x_h. The choice is $\mathbf{y}\gtreqless\mathbf{h}$ since $U\left(x_t,T_t;x_y,T_y\right)\gtreqless U\left(x_t,T_t;x_h,T_h\right)$

<u>*Axiom of transitivity*</u>: Assume the third displacement is $\mathbf{r}\in\mathbf{T}$ with x_r. If (**t.y**)>(**t.h**) and (**t.h**)>(**t.r**); the choice is (**t.y**)>(**t.r**) since $U\left(x_t,T_t;x_y,T_y\right)>U\left(x_t,T_t;x_h,T_h\right)>U\left(x_t,T_t;x_r,T_r\right)$. The same is true in the case of "<" and "=".

<u>*Axiom of addition*</u>: Assume the fourth displacement is $\mathbf{s}\in\mathbf{T}$ with x_s. If (**t.y**)>(**t.h**) and (**t.r**)>(**t.s**); the choice is (**t,y,r**)>(**t,h,s**) since $U\left(x_t,T_t;x_y,T_y;x_r,T_r\right)>U\left(x_t,T_t;x_h,T_h;x_s,T_s\right)$ The same is true in the case of "<" and "=".

Utility maximization: Supposing that *U* is a differentiable function of the combination of at least two economic goods defined in the economic good vector **x** and we consider all possible directional derivatives at a given point, in which of these directions does *U* change fastest and what is the maximum rate of change?

The gradient of the utility function at its initial position is ∇U(**x,T**) and the directional derivative (in direction δ) at this initial point is D(**x,T**,δ) = ∇U(**x,T**)′δ (the notation ′ denotes the transpose of a vector). This directional derivative indicates the slope of the utility function in the direction δ. Specifically, if the differential of the consumption combination decision is d(**x,T**) = δdα, where dα > 0, the differential utility change is dU(**x,T**)/dα = D(**x,T**,δ). The idea is that of the consumption combinations for unsystematic partial measures taken over a period of time determine the consumption directions, δ, making this directional derivative positive (see Raimondos-Moller and Woodland, 2015). In this respect, rather than searching for combinations with positive directional derivatives, we characterize the direction vector, δ, that maximizes the differential change in our objective. The locally optimal utility problem is expressed formally as

$$\max_{\delta}\left\{\nabla U\left(\mathbf{x},\mathbf{T}\right)'\delta\right\} \tag{17}$$

The solution for δ is, $\delta^{*}=\theta\,\nabla U\left(\mathbf{x},\mathbf{T}\right)$ and $\theta=\left|\nabla U\left(\mathbf{x},\mathbf{T}\right)\right|^{-1}$ for $\forall\theta>0$ where ∇U(**x,T**) is the gradient vector, whose elements are the partial derivatives ∂U(**x,T**)/∂(**x**$_i$,**T**$_i$), $i = 1, 2, \ldots, n$, and θ is the length of the gradient vector. In this solution, the maximum value of the direction derivative and the Euclidean length of the gradient vector occur when the unit vector has the same direction as ∇U(**x,T**). It is assumed that the best combination always yields improvement in the utility function.

APPENDIX II: Determination of the estimation restrictions

To isolate good demand, as a function of utility and total time use consists of time spent in working and consumption activities, both sides of the equation (4) are divided by $\sqrt[\varphi_i]{\Omega\prod T_w^{\alpha}\prod_i T_{c_i}^{\beta_i}}$ which gives

$$X_i=\Omega^{-1/\eta_i}\prod T_w^{\alpha/\eta_i}\prod_i T_{c_i}^{\beta_i/\eta_i}\prod_i U_i^{1/\eta_i} \tag{18}$$

Hypotheses 1: The goods and services consumed during the time spent in activities determine the satisfaction level of the individuals at time *t*. Therefore aggregate leisure time for all activities is equal to T^a (see eq.2.1), thus

$$S(U_i,T^a)=X_i=\Omega^{-1/\eta_i}\prod T_w^{\alpha/\eta_i}\prod_i T_{c_i}^{\beta_i/\eta_i}\prod_i U_i^{1/\eta_i} \tag{18.1}$$

The equation (18.1) represents the displacement of households' satisfactory state (i.e. *satisfaction*) at time *t* and at a utility distance *u* from one end *l* of the satisfaction string.

Proof 1: Thus, it is expected that the wave form of satisfaction function (18.1) satisfies the condition

$$S''_{T^a}=v^2S''_U \quad with \quad \forall 0<U<l;T_i^a>0 \tag{19}$$

Let *v* be the constant as the duration of the satisfactory state for a given period of time. This ratio determines the relation between utility and total time depending on density and the tension of the satisfaction string. By subjecting the equation (18.1), by assuming that $T^a=\prod T_w^{\alpha}\prod_i T_{c_i}^{\beta_i}$ and $\alpha+\beta=\theta$, to the aforementioned condition in (19) *v* can be isolated as

$$v=\left[\frac{\left(\frac{\theta^2}{\eta^2}+\frac{\theta}{\eta}\right)\prod_i U_i^{1/\eta_i}\prod_i T_{a_i}^{-\frac{\theta_i}{\eta_i}-2}}{\left(\frac{1}{\eta^2}+\frac{1}{\eta}\right)\prod_i \left(U_i^{1/\eta_i}\right)^{-2}\prod_i T_{a_i}^{-\frac{\theta_i}{\eta_i}}}\right]^{1/2} \tag{19.1}$$

Equation (19.1) could also be reduced to the form

$$U_i = v_i T_i^a = T_i^a \sqrt{\frac{1-\eta}{\theta(\theta+\eta)}} \tag{20}$$

where v is $\sqrt{1-\eta/\theta(\theta+\eta)}$. The limit exists for $D=\{(\theta,\eta)\,|\,\theta+\eta>0;\theta>1;1>\eta>0\}$ with $R=\{v\,|\,v(\eta,\theta)>0\}$. By using the stochastic variables of eq. (7) the eq. (20) leaves

$$\hat{U}_{ih} = T_{ih}^a \sqrt{\frac{1-\hat{\eta}_{ih}}{\left(\hat{\alpha}+\hat{\beta}\right)_{ih}\left(\hat{\alpha}+\hat{\beta}+\hat{\eta}\right)_{ih}}} \tag{21}$$

If the exponents of the Cobb-Douglas equation (4) are homogeneous to degree 1 then v =1. The number of oscillations for satisfaction for a specific consumption activity per unit of time is equal to the number of oscillations. The economic insight of this supposition is that every decision is always made at the end of the utility time period. This period ends up with the moment at which an individual's state of satisfaction falls back again to its initial level at time 0. For the sake of simplicity, as can be followed from equation (2.1), the degree of increase in satisfactory leisure time must always be equal to the total time degree of increases in working and consumption time; hence $\alpha+\beta=1$. Thus, the compact form of the equation (21) is

$$\hat{U}_{ih} = T_{ih}^a \sqrt{\frac{1-\hat{\eta}_{ih}}{1+\hat{\eta}_{ih}}} \tag{21.1}$$

Equation (21.1) gives the relation between T^a and $\hat{U}$ hence the satisfaction function as $\hat{S}(v,T^a)$. $\hat{\eta}_{ih}$ determines individual satisfaction level; hence the utility. The equation (21.1) implies that the satisfaction can be defined, *ceteris paribus*, as

$$D=\{\hat{\eta}_{ih}\,|\,0<\hat{\eta}_{ih}<1\} \text{ with } R=E(\hat{S}_{ih}=\hat{U}_{ih}\,|\,\hat{\eta}_{ih})=\begin{Bmatrix} 0 & \lim\limits_{\hat{\eta}_{ih}\to 1}\hat{S}_{ih} \\ 1 & \lim\limits_{\hat{\eta}_{ih}\to 0}\hat{S}_{ih}\end{Bmatrix} \tag{22}$$

The range of predicted satisfaction is defined as a decreasing relation while $\hat{\eta}_{ih}$ increases (*vice versa*).

<u>The value of</u> $\hat{\eta}_{ih}$:

Once regression function (7) has been carried out, the estimated parameters allow the calculation of $\hat{\eta}$ for each individual.

$$\hat{\eta} = \frac{\left(\tau_i - \hat{\varepsilon} - \hat{\delta}_0 - \hat{\delta}_1 \psi_i T_i^{c*} - \sum_{n=1}^{n} \hat{\delta}_{in} Z_{in}\right)\left(\beta T^{w*} - \alpha T_i^{c*}\right)}{\hat{\delta}_2 \psi_i T_i^{c*} T^{w*}} \tag{23}$$

This computation will be derived from the equation (7). Together with two aforementioned restrictions

$$\left.\begin{array}{l} \dfrac{\hat{\delta}_3}{\hat{\delta}_4} = \dfrac{\beta}{\alpha} \\ \beta + \alpha = 1 \end{array}\right\} \quad \alpha = \frac{\hat{\delta}_4}{\hat{\delta}_3 + \hat{\delta}_4}; \beta = \frac{\hat{\delta}_3}{\hat{\delta}_3 + \hat{\delta}_4} \tag{24}$$

The equation (23) becomes

$$\hat{\eta} = \frac{\left(\tau_i - \hat{\varepsilon} - \hat{\delta}_0 - \hat{\delta}_1 \psi_i T_i^{c*} - \sum_{n=1}^{n} \hat{\delta}_{in} Z_{in}\right)\left(\dfrac{\hat{\delta}_3 T^{w*} - \hat{\delta}_4 T_i^{c*}}{\hat{\delta}_3 + \hat{\delta}_4}\right)}{\hat{\delta}_2 \psi_i T_i^{c*} T^{w*}} \tag{25}$$

By replacing the equation (25) within the equation (21.1) $\hat{S}$ can be imputed for each individual.

Table A1: Descriptive statistics

Budget Shares	Variable	N	Mean	Std Dev	Minimum	Maximum
MONETARY EXPENDITURES(Temporal values)	Food	34357	0,1497	0,1371	0	0,8159
	Personal Care(with Health)	34357	0,0695	0,1228	0	0,9999
	Housing	34357	0,1739	0,1567	0	0,8808
	Clothing	34357	0,1368	0,2225	0	0,9999
	Education	34357	0,1216	0,2477	0	0,9991
	Transport	34357	0,1317	0,2102	0	1,0000
	Leisure	34357	0,0674	0,1318	0	0,9850
	Other	34357	0,1494	0,2380	0	0,9999
Budget Shares	Variable	N	Mean	Std Dev	Minimum	Maximum
TIME EXPENDITURES	Food	34357	0,0603	0,0080	0,0505	0,1205
	Personal Care(with Health)	34357	0,1931	0,0043	0,1585	0,2151
	Housing	34357	0,0477	0,0054	0,0361	0,0923
	Clothing	34357	0,0102	0,0046	0,0041	0,0377
	Education	34357	0,0071	0,0068	0,0009	0,0656
	Transport	34357	0,0759	0,0020	0,0579	0,0876
	Leisure	34357	0,4281	0,0186	0,2823	0,5313
	Other	34357	0,1774	0,0286	0,0185	0,3050
Budget Shares	Variable	N	Mean	Std Dev	Minimum	Maximum
FULL TIME EXPENDITURES	Food	34357	0,0587	0,0117	0,0022	0,1254
	Personal Care(with Health)	34357	0,1882	0,0328	0,0145	0,9007
	Housing	34357	0,0468	0,0093	0,0029	0,5460
	Clothing	34357	0,0190	0,0556	0,0006	0,7906
	Education	34357	0,0102	0,0133	0,0004	0,2054
	Transport	34357	0,0806	0,0479	0,0044	0,8717
	Leisure	34357	0,4139	0,0544	0,0270	0,8190
	Other	34357	0,1826	0,0657	0,0141	0,8611
Demographic characteristics:	Variable	N	Mean	Std Dev	Minimum	Maximum
	No. of children	34357	1,8356	1,8849	0	25
	Children smaller than age of 16	34357	0,9200	1,0308	0	12
	Children bigger than age of 16	34357	0,7963	1,1153	0	13
Education dummies:	Variable	N	Mean	Std Dev	Minimum	Maximum
	Without diploma	34357	0,0904	0,2868	0	1
	Primary education	34357	0,6125	0,4872	0	1
	Secondary education	34357	0,1893	0,3917	0	1
	Superior education	34357	0,1078	0,3101	0	1
Regional location dummies:	Variable	N	Mean	Std Dev	Minimum	Maximum
	Area (urban = 1)_Dummy	34357	0,6651	0,4720	0	1
Durables and luxury goods :	Variable	N	Mean	Std Dev	Minimum	Maximum
	Car	34357	0,2622	0,4398	0	1
	Television	34357	0,9775	0,1483	0	1
	Good heating system (includes central heating)	34357	0,1755	0,3804	0	1
	Cabel TV	34357	0,0373	0,1895	0	1
	Computer	34357	0,1215	0,3267	0	1
	Internet	34357	0,0426	0,2020	0	1
	Refrigerator	34357	0,9797	0,1409	0	1
	Deep freezer	34357	0,0411	0,1986	0	1
	Dish machine	34357	0,2219	0,4155	0	1
	Oven	34357	0,0496	0,2171	0	1
	Clima	34357	0,0385	0,1924	0	1
	Cell phones	34357	0,6761	0,4679	0	1
Housing:	Variable	N	Mean	Std Dev	Minimum	Maximum
	Home ownership	34357	0,6674	0,4711	0	1
	Number of rooms in the house	34357	3,4383	0,7676	1	12
	Owing house-resting debt	34357	0,0271	0,1624	0	1
	Rented accommodation	34357	0,2488	0,4323	0	1

Table A2: Glossary of variables

T_i : Time assigned to activity *i*

W : Time assigned to work

L : Time assigned to leisure

t_i : Time assigned to travel (mode *i*)

t : Exogenous travel time

c_i : Travel cost (mode *i*)

c : Travel cost

Z_i : Final good *i*

f_i : Production function of commodity *i*

P_i : Price of good *i*

X_i : Consumption of good *i*

P_W : Price of goods associated with the work activity

X_W : Consumption of goods associated with work activity

w_i : Money reward of activity *i*

w_L : Money reward of leisure

w : Wage rate (work)

G : Aggregate consumption in money units

I_f : Individual's fixed income

T : Total time available

U : Utility function

F : Function that accounts for the limitations imposed by the institutional setting within which employment opportunities are encountered

s : Schedule time (a specific time of the day)

μ : Lagrange multiplier of time restriction

λ : Lagrange multiplier of income restriction

ν : Lagrange multiplier of schedule restriction

κ_i : Lagrange multiplier of minimum time requirement of activity *i*

b_{ij} : Minimum time requirement of activity i per unit of activity j

τ_i : Unit of time assigned to per consumption of good i $\left(T_i / X_i\right)$

v : Other incomes

Table A3: Heckman two step selection model results for 2006

Natural logarithm of monthly salary

Variable	Male		Female	
Age	0.267	***	0.405	***
	(0.027)		(0.098)	
Age Squared	-0.012	***	-0.021	***
	(0.001)		(0.006)	
Primary Education	0.484	***	0.343	***
	(0.028)		(0.098)	
Secondary Education	0.827	***	1.465	***
	(0.035)		(0.128)	
Tertiary Education	1.193	***	2.104	***
	(0.040)		(0.123)	
Area (Urban)	0.250	***	0.309	***
	(0.017)		(0.072)	
Couple	0.490	***	5.300	***
	(0.035)		(0.050)	
Number of Children	0.713	***	0.279	***
	(0.010)		(0.088)	
Household Size	- 0.196	***	- 0.186	***
	(0.041)		(0.017)	
Constant	4.651	***	3.184	***
	(0.118)		(0.495)	
Error terms correlation rho	0.130	***	-0.162	*
	(0.034)		(0.153)	
λ	0.081	***	0.081	***
	(0.021)		(0.021)	
Log pseudo likelihood	-1231		-2976	
No. of obs.	12875		12033	
Number of Workers	6094		936	

***p<0.01, **p<0.05, *p<0.1

Table A4: Parameter estimates time assignment equations for all commodities (GMM)(Whole Population, years between 2003-2006)

Variables	Food	t- ratio	Pc+Health	t- ratio	Housing	t- ratio	Clothing	t- ratio	Education	t- ratio	Transport	t- ratio	Leisure	t- ratio	Others	t- ratio
Constant	0,05767	134,97	0,18975	101,39	0,04630	140,23	0,00700	3,51	0,00526	9,25	0,07490	30,74	0,41732	137,26	0,20565	64,82
2003	-		-		-		-		-		-		-			
2004	0,00011	2,24	0,00040	4,76	0,00047	14,07	-0,00025	-0,98	0,00001	0,44	-0,00004	-0,29	-0,00126	-5,09	-0,00292	-11,08
2005	-0,01206	-165,13	-0,02687	-145,69	-0,00966	-167,91	0,04766	118,18	0,01786	243,46	0,02323	77,22	-0,07357	-163,54	0,03117	68,84
2006	0,01825	136,45	-0,00051	-2,91	0,01237	92,28	0,01071	38,13	0,01361	63,51	0,00246	14,34	-0,01231	-19,68	-0,04688	-62,79
Good heating system	-0,00003	-0,23	-0,00006	-0,16	0,00033	3,33	0,00241	2,58	-0,00050	-2,29	-0,00227	-3,39	-0,00060	-0,75	-0,00053	-0,47
Number of rooms in the house	0,00009	1,72	-0,00001	-0,03	0,00019	4,48	0,00005	0,14	-0,00012	-1,41	-0,00017	-0,56	-0,00007	-0,2	-0,00029	-0,76
Home ownership	-0,00123	-16	0,00029	1,13	0,00001	0,09	-0,00130	-2,4	0,00000	0	0,00013	0,32	0,00116	2,35	0,00088	1,46
Area (urban = 1)	-0,00133	-15,8	0,00202	9,57	-0,00369	-47,9	0,00056	1,44	0,00179	14,09	0,00267	7,44	-0,00104	-2,2	-0,00084	-1,56
Computer	0,00082	4,74	-0,00053	-0,92	0,00061	4,31	0,00068	0,51	0,00017	0,52	-0,00227	-2,44	0,00138	1,2	-0,00057	-0,34
Car	0,00027	3,04	0,00082	2,48	0,00038	5,28	-0,00059	-0,86	0,00023	1,86	-0,00602	-16,94	0,00320	5,52	0,00140	1,78
Without diploma	-0,00254	-15,46	0,00223	4,06	-0,00186	-14,2	0,00067	0,68	-0,00376	-16,58	-0,00077	-0,92	0,00377	3,72	0,00037	0,3
Primary Education	-0,00112	-9,14	-0,00034	-1	-0,00103	-10,61	0,00173	2,3	-0,00205	-11,49	-0,00002	-0,04	-0,00079	-1,1	0,00173	1,75
Secondary Education	-0,00005	-0,39	-0,00076	-1,96	-0,00033	-3,06	-0,00009	-0,1	-0,00145	-6,54	0,00041	0,56	-0,00144	-1,68	0,00284	2,55
Television	0,00048	2,13	-0,00061	-0,58	0,00036	2,02	0,00122	1,17	0,00041	1,26	0,00011	0,09	-0,00269	-1,74	0,00146	0,85
Internet	0,00108	4,33	0,00228	3,16	0,00111	5,28	-0,00075	-0,36	-0,00023	-0,46	-0,00008	-0,06	0,00418	2,63	-0,00872	-3,95
Refrigerator	-0,00012	-0,42	0,00038	0,22	0,00013	0,53	0,00106	0,81	-0,00029	-0,75	0,00123	0,64	0,00174	0,82	-0,00389	-1,79
Dish Machine	0,00053	4,22	0,00057	1,41	0,00053	5,47	-0,00012	-0,14	-0,00018	-0,84	0,00000	-0,01	0,00311	3,91	-0,00355	-3,13
Oven	-0,00034	-1,91	-0,00075	-1,38	-0,00011	-0,72	-0,00240	-1,84	-0,00035	-1,27	0,00165	1,7	-0,00250	-2,17	0,00438	2,68
Cell phone	0,00351	42,71	0,00129	3,89	0,00245	36,92	0,00045	0,81	0,00013	1,19	-0,00095	-1,85	0,00414	7,69	-0,00983	-16,08
Cablecast	-0,00069	-3,39	-0,00056	-0,98	0,00013	0,77	0,00240	1,67	0,00094	2,58	-0,00262	-2,6	0,00527	3,87	-0,00253	-1,39
Summer house ownership	-0,00023	-0,63	0,00228	2,05	0,00050	1,67	0,00039	0,14	-0,00125	-1,98	0,00026	0,24	0,00456	1,89	-0,00836	-2,32
Sex	-0,00026	-1,44	0,00048	0,78	0,00021	1,57	-0,00135	-1,42	0,00110	4,59	0,00145	1,43	0,00665	5,55	-0,00909	-6,44
$\psi T c$	-0,00003	-36,49	0,00003	7,25	0,00000	-4,29	0,00004	7,79	-0,00002	-36,98	0,00006	12,7	0,00053	29,72	-0,00023	-19,35
$\psi T c T w$	0,00000	-4,19	-0,00003	-18,35	-0,00001	-15,64	0,00000	4,97	0,00000	-0,02	-0,00003	-10,07	-0,00043	-35,56	0,00011	20,53
$T w$	0,11140	7417,83	0,51329	10610000	0,52505	19,79	0,04195	4414,69	0,39166	3656,97	0,56394	59,97	0,72800	521,73	0,51433	753,99
$T c$	0,88860	717,87	0,48671	15110000	0,47495	15,18	0,95805	5420,37	0,60834	5599,2	0,43606	96,81	0,27200	292,16	0,48567	601,32

Sargan Statistics (overidentification test of all instruments= 0,430 with chi-square P value =0,8063> 0,05

APENDIX III: Total utility

The equation (21.1) also represents a one-dimensional form for the scalar sinusoidal (harmonic) moving wave function as follows.

$$s(\hat{U},T^{a})= f(U-T^{a}\sqrt{\frac{1-\hat{\eta}}{1+\hat{\eta}}}) \tag{26}$$

Satisfaction wave function *s(U, T)* indicates the *U* coordinate—the transverse position—of any element located at position utility *u* at any satisfactory leisure time *t*. Here, *v* is the speed of a satisfaction wave function equal to $\sqrt{1-\hat{\eta}/1+\hat{\eta}}$.

Different goods and services may be represented in the consumption set as durable or non-durable used over a given period, and hence different utilities are felt from each consumption. Total utility could be obtained by superposition of the satisfaction waves with different frequencies, amplitudes with the phase constant in the short term for these expenditure groups. In this sense, the absolute value of the maximum displacement from a state of equilibrium for a utility of the medium gives the amplitude *A* for each satisfactory wave measured by $A_{ih}=|\hat{U}_{ih}-\bar{U}_{i}|$, where $\bar{U}$ is the mean of i^{th} consumption group. In the form of sinusoidal wave motion, *A* is the ratio between the frequencies over distances traveled by each utility wave. The length of these satisfaction waves are the distance from one crest (or troughs) to the next. Consumption decision exists whenever utility falls back to the minimum level of utility for any given wave function. This is the minimum utility point (i.e. trough) within the total satisfaction period which could also be measured by the curvature function.

Hypothesis 2: The utility has a specific emotional value instantly felt at any point on the satisfaction string.

Proof 2: The equation (21.1) allows us to see how quickly utility changes direction at any point in time depending on a given satisfaction level. This could be observed through measuring the magnitude of the rate of change of the unit tangent vector with respect to arc length. Thus, it could be argued that the smooth changes on the satisfaction curve could give information about how an individual's emotional state changes at any point in time. Such an instant utility function could be obtained from the curvature equation applied on the two dimensional parabola functions[14], after taking the square of both sides of the equation (21.1) as

$$K_{\hat{U}_{ih}}(T_{ih}^{a})=\frac{2(1-\hat{\eta}_{ih})(1+\hat{\eta}_{ih})^{2}}{\left[T_{ih}^{a}2(1-\hat{\eta}_{ih})+1+\hat{\eta}_{ih}\right]^{3}} \tag{27}$$

The curvature curve is continuous for D: $\left\{\left(\hat{\eta},T^{a}\right)\in\mathbb{R}^{2}:2x(\hat{\eta}-1)\neq\hat{\eta}+1\right\}$ with $\hat{\eta}\in(0,1)$. The equation (27) is obtained by tangent points on the satisfaction string and this curve equation allows knowing zero emotional chances (i.e. being neutral for one extra unit of consumption) at a point of time where the consumption stops. All maximum and minimum points on that curve satisfy the first order condition for curvature function (27) at

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[14] This is $\kappa_{\hat{U}}(T)=\frac{\left|U_{T_{ih}}{''}\right|}{\left[1+(U_{T_{ih}}{'})^{2}\right]^{3/2}}$

$$\frac{\partial K_{\hat{U}}}{\partial T^a} = -\frac{12(\hat{\eta}^2 - 1)^2}{(-2T^a(\hat{\eta} - 1) + \hat{\eta} + 1)^4} \text{ with } \hat{\eta} \in (0,1) \text{ and } T^a > 0 \tag{28}$$

In order to obtain the solution of equation (28), let T_λ^a be the adjacent periodic thought or crest as the wave length which is equal to

$$T_\lambda^a = \frac{\sqrt[4]{-\frac{12(\hat{\eta}^2 - 1)^2}{K_{\hat{U}}}} - \hat{\eta} - 1}{-2(\hat{\eta} - 1)} \tag{28.1}$$

Total Utility

Once the wave length is obtained, we get the angular wave number $k = 2\pi / T_\lambda^a$ and angular frequency $\omega_{an} = 2\pi / T^a$. The frequency $1/T^a$ is computed by the period for each consumption group. According to the superposition principle the net is sum of the individual displacement. The wave equation is linear. More precisely, we superpose 8 satisfaction waves with different amplitudes and frequencies.

$$s(\hat{U}, T^a) = \sum_{i=1}^{8} A_i \sin(k_i U_{ih} - \overline{\omega}_i T_{ih}^a + \mu_{ih}) \tag{29}$$